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Melting temperatures of nucleic acids: Discrepancies in analysis

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Abstract

Melting temperature, $T_{\rm m}$, is an important property of nucleic acid duplexes. It is typically determined from spectroscopic or calorimetric melting experiments. More than one analytical method has been used to extract $T_{\rm m}$ values from experimental melting data. Unfortunately, different methods do not give the same results; the same melting data can be assigned different $T_{\rm m}$ values depending upon which method is used to process that data. Inconsistencies or systematic errors between $T_{\rm m}$ s reported in published data sets can be significant and add confusion to the field. Errors introduced from analysis can be greater than experimental errors, ranging from a fraction of degree to several degrees. Of the various methods, the most consistent and meaningful approach defines melting temperature as the temperature at the transition midpoint where half of the base pairs are melted and standard free energy is zero. Assuming a two-state melting behavior, we present here a set of general equations that can be used to reconcile these analytical $T_{\rm m}$ differences and convert results to the correct melting temperatures at the transition midpoint. Melting temperatures collected from published sources, which were analyzed using different methods, can now be corrected for these discrepancies and compared on equal footing. The similar corrections apply to $T_{\rm m}$ differences between calorimetric and spectroscopic melting curves. New algorithm for selection of linear sloping baselines, 2nd derivative method, is suggested, which can be used to automate melting curve analysis. © 2005 Elsevier B.V. All rights reserved.

Keywords: DNA; RNA; Melting curve; Melting profile; Thermodynamic analysis

1. Introduction

Melting temperature, $T_{\rm m}$, is a physical property of nucleic acids that gives information about the stability of duplexes in a specified environment. $T_{\rm m}$ values are useful in a variety of fields ranging from practical assay design in molecular biology to theoretical biophysics. We and others [1] have been collecting and importing published thermodynamic data including melting temperatures into databases. Large, statistically significant sets of experimental melting data from independent sources are very useful for development and refinement of models that predict stability and hybridization energetics of nucleic acids. Unfortunately, published $T_{\rm m}$ values are not consistent; results vary between different laboratories. These discrepancies arise because of experimental errors in data collection and differences between

2. Theoretical results

Melting profiles are usually collected using spectroscopic experiments. The intensity signal (absorbance, fluorescence, molar ellipticity) is measured as a function of temperature. In the following, we develop a theoretical model using an ultraviolet absorbance versus temperature profile as an

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methods of data analysis [2,3]. Different analytical methods provide different $T_{\rm m}$ values from the same experimentally measured melting curves, i.e., inherent systematic differences in reported melting temperatures exist. Building upon previous works [2–7], we present here a set of general formulas that allow correction of reported $T_{\rm m}$ values to the transition midpoint $T_{\rm m}$ where half of the base pairs are broken and allow different data set to be integrated. In addition, a novel method for automatic selection of sloping baselines and some advantages of plotting ${\rm d}\theta/{\rm d}(1/T)$ derivative melting curves are described.

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example. However, the same formalism will apply to other spectroscopic measurements, including circular dichroism, fluorescence or NMR melting experiments. Fig. 1 shows a typical ultraviolet melting profile and its derivatives for 10 base pair long DNA duplex [8]. Panel A displays dependence of absorbance at 268 nm on sample temperature. The upper and lower baselines are established from the linear regions before and after the transition. The fraction of melted base pairs, θ , is calculated from the standard formula, $\theta = (A - A_{\rm L})/(A_{\rm U} - A_{\rm L})$, where A, $A_{\rm L}$, and $A_{\rm U}$ are sample absorbance, absorbance of the lower baseline, and absorbance of the upper baseline, respectively [9]. The resulting melting profile (Fig. 1B) is usually smoothed by a digital filter [13] or by fitting a polynomial to a sliding window of data points. The correct definition of melting temperature, $T_{\rm m}$ (half), is the temperature at the midpoint of transition where $\theta = 0.5$ and standard free energy of transition is zero. Assuming that spectroscopic signal is proportional to the number of melted base pairs, $T_{\rm m}$ (half) is a temperature where half of the base pairs are melted from the total number of base pairs in the nucleic acid sample. For the duplex on Fig. 1, $T_{\rm m}$ (half) of 55.4 °C was determined.

In common practice, alternative analytical methods are sometimes employed, which yield different $T_{\rm m}$ values [2,3]. These analytical methods are often available as a part of software package supplied with spectrophotometers. Derivatives of θ with respect to temperature or reciprocal of the temperature are calculated (panels C and D on Fig. 1). Melting temperatures, $T_{\rm m}(\theta - T)$ and $T_{\rm m}(\theta-1/T)$ are read as temperatures of peak height maxima of $d\theta/dT$ and $d\theta/d(1/T)$ derivative melting profiles, respectively. These two methods essentially determine melting temperatures from the inflection point of an S-shaped melting curve. Because melting curves are not exactly symmetrical, $T_{\rm m}({\rm half})$, $T_{\rm m}(\theta-T)$, and $T_{\rm m}(\theta-1/T)$ are not identical. Differences among these values exist, typically $T_{\rm m}(\theta - 1/T) > T_{\rm m}(\theta - T) > T_{\rm m}({\rm half})$, as is illustrated in Fig. 1. When spectroscopic equipment is tested and validated, and proper experimental procedures are followed, experimental errors of melting temperatures that stem from data collection can be as low as 0.2 °C [3,8,10]. Differences among $T_{\rm m}({\rm half})$, $T_{\rm m}(\theta-T)$ and $T_{\rm m}(\theta-1/T)$ are outside of these experimental errors, e.g., the difference $T_{\rm m}(\theta-1/T)-T_{\rm m}({\rm half})$ is equal to 1.5 °C for the melting profile shown in Fig. 1. Therefore, it would be useful to correct published $T_{\rm m}(\theta-T)$ and $T_{\rm m}(\theta-1/T)$ temperatures to $T_{\rm m}$ (half) values, so that data from different laboratories can be properly compared.

Consider bimolecular hybridization reaction of two different non-self-complementary oligomers S_1 and S_2 of the same concentrations $C_t/2$. The oligomers anneal to form a duplex D,

$$S_1 + S_2 \rightleftharpoons D$$

Assuming that the transition proceeds in two-state fashion, i.e., no significant population of partially melted

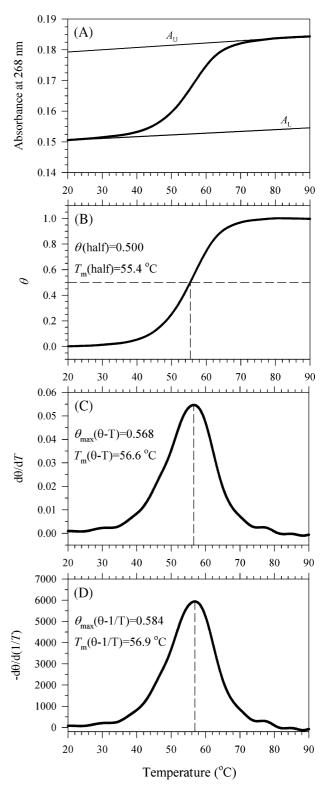


Fig. 1. Experimental ultraviolet melting profile of d(TGGCGAGCAC) duplex oligomer (A) was analyzed using three different methods: fraction of broken (melted) base pairs was plotted vs. temperature (B). First derivatives ${\rm d}\theta/{\rm d}T$ (C) and ${\rm d}\theta/{\rm d}(1/T)$ (D) were plotted vs. temperature. The DNA duplex was dissolved in 1M NaCl-phosphate buffer, at C_t =2 μ M, pH 7.0 [8]. Melting profiles were averaged from four heating and four cooling melting curves.

duplexes is detected, the fraction of *melted base pairs* is equal to fraction of *melted duplexes* and can be written as,

$$\theta = \frac{[S_1] + [S_2]}{C_t} \tag{1}$$

where the total single strand concentration, $C_t = [S_1] + [S_2] + 2[D]$. Note that the equations are sometimes cast in terms of fraction of intact duplexes [2,4,7], α , which is simply $1 - \theta$. The equilibrium constant of annealing reaction [2,4,7] in terms of θ is,

$$K_{\rm a} = \frac{[D]}{[S_1][S_2]} = \frac{2(1-\theta)}{\theta^2 C_{\rm t}}.$$
 (2)

Solution of this equation gives us the formula for prediction of melting curves (Fig. 1B),

$$\theta = \frac{\sqrt{1 + 2K_aC_t} - 1}{K_aC_t}.$$
 (3)

Substitution of K_a from the Eq. (2) into van't Hoff Eq. (4),

$$\frac{\mathrm{dln}K_{\mathrm{a}}}{\mathrm{d}T} = \frac{\Delta H^{\circ}}{RT^{2}} \tag{4}$$

and further differentiation leads to the equation for the derivative melting curves, e.g., the graph in Fig. 1C,

$$\frac{\mathrm{d}\theta}{\mathrm{d}T} = -\frac{\Delta H^{\circ}}{RT^{2}} \frac{\theta(1-\theta)}{2-\theta}.$$
 (5)

This expression relates $\mathrm{d}\theta/\mathrm{d}T$ to ΔH° at any θ . To find the fraction of melted base pairs at maximum of this derivative melting curve, $\theta_{\mathrm{max}}(\theta-T)$, the second derivative is calculated,

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}T^2} = \frac{\Delta H^{\circ}}{RT^3} \frac{\theta(1-\theta)}{2-\theta} \left(2 + \frac{\Delta H^{\circ}}{RT} \frac{\theta^2 + 2(1-2\theta)}{\left(2-\theta\right)^2} \right). \quad (6)$$

Solution of the Eq. (6) for θ when $d^2\theta/dT^2=0$ and $0<\theta<1$ yields,

$$\theta_{\text{max}}(\theta - T) = 2 - \sqrt{\frac{2}{1 + 2\frac{RT}{\Delta H^{\circ}}}} = 2 - \sqrt{\frac{2}{1 + 2B}}$$
 (7)

where B denotes the ratio of $RT/\Delta H^{\circ}$, which is often present in equations. Since negative ΔH° values are observed for strand annealing reactions, the values of B are negative.

Analogous procedure can be employed to get a relationship for $\theta_{\rm max}(\theta-1/T)$ at the maximum of ${\rm d}\theta/{\rm d}(1/T)$ derivative melting curve. For bimolecular equilibria, the resulting equations are summarized in Table 1. Two important features are observed. (1) Comparing hybridization of a self-complementary strand with hybridization of non-self-complementary strands, equations for the equilibrium constant are different for each type of hybridizations (row 1 of Table 1). However, equations for derivative melting curves and their maxima are identical for both types

Table 1 Thermodynamic models of bimolecular equilibria between two single strands, S_1 , S_2 , and the duplex D, where both single strands have the same concentration

| concentration | | | | | | |
|---------------------------------|--|--|--|--|--|--|
| Reaction type | $S_1 + S_2 \rightleftharpoons D$ (non-self-complementary) | $2S \rightleftharpoons D$ (self-complementary) | | | | |
| Equilibrium constant | $K_{a} = \frac{[D]}{[S_1][S_2]} = \frac{2(1-\theta)}{\theta^2 C_{t}}$ | $K_{\rm a} = \frac{[D]}{[S]^2} = \frac{(1-\theta)}{2\theta^2 C_{\rm t}}$ | | | | |
| Melting curve | $\theta = \frac{\sqrt{1 + 2K_{a}C_{t}} - 1}{K_{a}C_{t}}$ $d\theta / dT \text{ derivative melting cur}$ | $\theta = \frac{\sqrt{1 + 8K_aC_t} - 1}{4K_aC_t}$ rves | | | | |
| First derivative | $\frac{\mathrm{d}\theta}{\mathrm{d}T} = -\frac{\Delta H^{\circ}}{RT^2} \frac{\theta(1-\theta)}{(2-\theta)}$ | | | | | |
| Second derivative | $\frac{\mathrm{d}^2\theta}{\mathrm{d}T^2} = \frac{\Delta H^{\circ}}{RT^3} \frac{\theta(1-\theta)}{(2-\theta)} \left(2 + \frac{\Delta H^{\circ}}{RT} \frac{\theta^2 + 2(1-2\theta)}{(2-\theta)^2} \right)$ | | | | | |
| $\theta_{\max}(\theta - T)^{a}$ | $\theta_{\text{max}}(\theta - T) = 2 - \sqrt{\frac{2}{1 + 2B}}$ | | | | | |
| First derivative | $\frac{\mathrm{d}\theta/\mathrm{d}(1/T)}{\mathrm{d}(1/T)} = \frac{\Delta H^{\circ}}{R} \bullet \frac{\theta(1-\theta)}{2-\theta}$ | g cui ves | | | | |
| Second derivative | $\frac{\mathrm{d}^2 \theta}{\mathrm{d}(1/T)^2} = \left(\frac{\Delta H^{\circ}}{R}\right)^2 \frac{\theta (1-t)^2}{2-t^2}$ | $\frac{\theta}{\theta} \frac{\theta^2 + 2(1 - 2\theta)}{(2 - \theta)^2}$ | | | | |
| $\theta_{\max}(\theta - 1/T)$ | $\theta_{\max}(\theta - 1/T) = 2 - \sqrt{2}$ | | | | | |

of hybridizations (rows 3–8 of Table 1). In other words, derivative melting curves are described by the same relationships, in terms of θ , for hybridizations of both identical and non-identical single strands. (2) Furthermore, the value of θ at the maximum of $\mathrm{d}\theta/\mathrm{d}(1/T)$ derivative melting curve is independent of ΔH° and therefore is also independent of base sequence and oligomer length. The $\theta_{\mathrm{max}}(\theta-1/T)$ is constant and equal to 0.5858 for bimolecular hybridizations when single strands are at the same concentration

Equations from Table 1 can be used to calculate correct melting temperatures, $T_{\rm m}({\rm half})$, from published $T_{\rm m}(\theta-T)$, $T_{\rm m}(\theta-1/T)$ values. Temperatures can be estimated from transition enthalpy, ΔH° , entropy, ΔS° , and equilibrium constants

$$T_{\rm m}({\rm half}) = \frac{\Delta H^{\rm o}}{\Delta S^{\rm o} - R {\rm ln} K_{\rm a}({\rm half})} \tag{8}$$

$$T_{\rm m}(\theta - T) = \frac{\Delta H^{\circ}}{\Delta S^{\circ} - R \ln K_{\rm a}(\theta - T)}$$
(9)

where $K_a(\text{half})$ and $K_a(\theta - T)$ are the equilibrium constants of hybridization reaction at $T_m(\text{half})$ and $T_m(\theta - T)$ temperatures, respectively. Combining Eqs. (8) and (9) yields,

$$T_{\rm m}({\rm half}) = \frac{\Delta H^{\circ}}{\frac{\Delta H^{\circ}}{T_{\rm m}(\theta - T)} + R \ln \left[\frac{K_{\rm a}(\theta - T)}{K_{\rm a}({\rm half})}\right]}$$
(10)

which represents a general equation that allows calculation of $T_{\rm m}({\rm half})$ from the temperature at maxima of derivative melting curve, $T_{\rm m}(\theta-T)$. The analogous equation was found for the ${\rm d}\theta/{\rm d}(1/T)$ derivative melting curves,

$$T_{\rm m}({\rm half}) = \frac{\Delta H^{\circ}}{\frac{\Delta H^{\circ}}{T_{\rm m}(\theta - 1/T)} + R \ln \left[\frac{K_{\rm a}(\theta - 1/T)}{K_{\rm a}({\rm half})}\right]} \tag{11}$$

which can be applied to calculate $T_{\rm m}$ (half) melting temperatures from $T_{\rm m}(\theta-1/T)$ values. To complete the calculation of $T_{\rm m}$ (half), ΔH° and $K_{\rm a}$ must be known. If the value of ΔH° is not available experimentally, it can be estimated from the nearest-neighbor model [11]. The equilibrium constants, which enter into the Eqs. (10) and (11), are calculated from the $\theta_{\rm max}(\theta-T)$, $\theta_{\rm max}(\theta-1/T)$ values and from the $K_{\rm a}$ relationships shown in row 1 of Table 1. For example, let us evaluate equilibrium constants for hybridization of non-self-complementary single strands. At $T_{\rm m}$ (half), the value of θ is 0.5 by definition. Therefore, the equilibrium constant at $T_{\rm m}$ (half) is evaluated as,

$$K_{\rm a}({\rm half}) = \frac{2(1-0.5)}{0.5^2 C_{\rm t}} = \frac{4}{C_{\rm t}}.$$
 (12)

Eq. (7) can be incorporated into Eq. (2) to give the equilibrium constant at $T_{\rm m}(\theta-T)$,

$$K_{a}(\theta - T) = \frac{\sqrt{2(1+2B)} - (1+2B)}{C_{t}[3+4B-2\sqrt{2(1+2B)}]}$$
$$= \frac{1-2B-8B^{2}+\sqrt{2(1+2B)}}{C_{t}(1+4B)^{2}}$$
(13)

The expression for $\theta_{\text{max}}(\theta - 1/T)$ from Table 1 can be substituted into Eq. (2) to give the equilibrium constant at $T_{\text{m}}(\theta - 1/T)$,

$$K_{\rm a}(\theta - 1/T) = \frac{1 + \sqrt{2}}{C_{\rm t}}$$
 (14)

The resulting $K_{\rm a}$ values from Eqs. (12)-(14) are then used with Eqs. (10) and (11) to calculate corrected melting temperatures, $T_{\rm m}$ (half). Note that the Eqs. (12)-(14) were derived assuming hybridization of non-self-complementary strands. If the annealing strands are self-complementary (identical), the appropriate equation for $K_{\rm a}$ given in row 1 of Table 1 must be used to obtain expressions for $K_{\rm a}$ (half), $K_{\rm a}(\theta-T)$ and $K_{\rm a}(\theta-1/T)$.

Hybridization reactions of nucleic acids are not limited to bimolecular systems. Hairpins, 3-way junctions and 4-way junctions fold into structures, which consist of different numbers of single strands. The relationships between equilibrium constants and fractions of melted base pairs can be generalized for n-strand hybridizations [5,7]. Assuming that single strands are present in the same concentration, $C_{\rm t}/n$, we derived general equations, which can be employed to predict derivative melting profiles, melting temperatures and their discrepancies. The equations

for hybridization reactions of any molecularity are presented in the Table 2 and were derived from relationships between K_a and θ using procedures similar to that previously shown for bimolecular hybridizations. Comparison of Table 1 with Table 2 reveals that the expressions are consistent. The equations in Table 2 reduce to the equations of Table 1 when n equal to 2 is considered. Two features that were observed for bimolecular hybridizations are also seen for general *n*-strand hybridizations. Again, derivative melting curves are described by identical expressions for both selfcomplementary and non-self-complementary single strands. The equations in rows 2-7 of Table 2 hold for both cases. Further, the values of θ at maximum of $d\theta/d(1/T)$ derivative melting curves are independent of oligomer sequence and length. They depend solely on molecularity of the hybridization reaction. Corrected $T_{\rm m}$ (half) values are predicted in a manner similar to bimolecular hybridizations. General Eqs. (10) and (11) can be used, because they hold for hybridizations of any molecularity. Equilibrium constants, which enter the Eqs. (10) and (11), need to be evaluated from equations given in the first row of Table 2 and corresponding $\theta_{\text{max}}(\theta - T)$, $\theta_{\text{max}}(\theta - 1/T)$ values.

Table 2 Thermodynamic models of general n-molecular equilibria between the single strands, S_1 , S_2 , ... S_n , and the helical complex, D_n , when all single strands are either non-self-complementary or self-complementary

| Reaction type | $S_1 + S_2 + \dots S_n \rightleftharpoons D_n$ (non-self-complementary) | $nS \rightleftharpoons D_n$ (self-complementary) |
|--------------------------------------|---|---|
| Equilibrium constant ^a | $K_{a} = \frac{[D_{n}]}{[S_{1}][S_{2}][S_{n}]}$ $= \frac{n^{n-1}(1-\theta)}{\theta^{n}C^{n-1}}$ | $K_{a} = \frac{[D_{n}]}{[S]^{n}}$ $= \frac{(1 - \theta)}{n\theta^{n}C^{n-1}}$ |
| First derivative | $\frac{d\theta}{dT} = -\frac{\Delta H^o}{RT^2} \frac{\theta(1-\theta)}{\theta + n(1-\theta)}$ | no C_{t} |
| Second derivative | $\frac{\mathrm{d}^2 \theta}{\mathrm{d}T^2} = \frac{\Delta H^{\circ}}{RT^3} \frac{\theta (1-\theta)}{\theta + n(1-\theta)}$ | |
| $\theta_{\rm max}(\theta-T)^{\rm b}$ | $\times \left(2 + \frac{\Delta H^{\circ}}{RT} \frac{\theta^{2}(n-1)}{\left[\theta + n(\theta)\right]} \right)$ $\theta_{\max}(\theta - T) = \frac{1}{n-1} \left[n - \sqrt{\frac{1}{1+\theta}}\right]$ | |
| First derivative | $\frac{d\theta/d(1/T)}{d\theta} = \frac{\Delta H^{\circ}}{R} \frac{\theta(1-\theta)}{\theta + n(1-\theta)}$ | urves |
| Second derivative | $\frac{\mathrm{d}^2 \theta}{\mathrm{d}(1/T)^2} = \left(\frac{\Delta H^{\circ}}{R}\right)^2 \frac{\theta (1 - \theta)^2}{\theta + n(1 - \theta)^2}$ | $\frac{1}{(\theta)}$ |
| $\theta_{\text{max}}(\theta - 1/T)$ | $\times \frac{\theta^{2}(n-1) + n(1-\theta)}{\left[\theta + n(1-\theta)\right]}$ $\theta_{\max}(\theta - 1/T) = \frac{1}{n-1} \left(n - \sqrt{n}\right)$ | |

^a Each strand has the same concentration of C_t/n . $(C_t=[S_1]+[S_2]+\ldots+[S_n]+n[D_n])$.

b $(B = RT_{\rm m}/\Delta H^{\circ})$

Table 3 shows expressions for two special cases of nucleic acid hybridizations. The left side of the table presents equations for hairpin-random coil equilibrium when a single strand folds into an intramolecular structure. The right side of Table 3 presents equations for a bimolecular reaction when one of two single strands (S_1) has a significantly higher concentration than the other strand (S_2) . Under this condition, the expression for K_a of bimolecular reaction can be simplified and resembles the equation for K_a of hairpin-coil equilibrium (row 1 of Table 3). Further differentiation demonstrates that the expressions describing the derivative melting curves are same for both equilibria (row 3-8 of Table 3). It has been long recognized [3,4] that a plot of $d\theta/d(1/T)$ versus temperature for intramolecular hairpin-coil equilibrium has a maximum exactly at the midpoint of melting transition, i.e., $\theta_{\text{max}}(\theta - 1/T)$ is equal to 0.5 and in turn $T_{\text{m}}(\theta - 1/T)$ is equal to $T_{\rm m}$ (half). The analysis in Table 3 shows that this is also true for the bimolecular equilibrium when one of the single strands is in significant excess (at least $10\times$ or more). Interestingly, this kind of equilibrium is seen in some important molecular biology applications, e.g., detection of gene targets using oligomeric probes. We conclude that in these two specific cases of nucleic acid equilibria, $T_{\rm m}$ (half) can be read directly as a temperature at the peak height maximum of $d\theta/d(1/T)$ derivative melting curve.

Table 3 Comparison of hairpin-melted coil equilibria to bimolecular equilibria where the analytical concentration (C_1) of the single strand S_1 is in significant excess to the concentration (C_2) of single strand S_2

| Reaction type | $S \rightleftharpoons D$ | $S_1 + S_2 \rightleftharpoons D$ | | | |
|--------------------------------------|--|--|--|--|--|
| | (melted coil-hairpin) | (non-self-complementary | | | |
| | | strands and $C_1 >> C_2$) | | | |
| Equilibrium constant | $K_{\rm a} = \frac{[D]}{[S]} = \frac{1-\theta}{\theta}$ K | | | | |
| | | $= \frac{1-\theta}{\theta \left[C_1 - C_2(1-\theta) \right]} \approx \frac{1-\theta}{\theta C_1}$ | | | |
| Melting curve | $\theta = \frac{1}{1 + K_{\rm a}}$ | $\theta \approx \frac{1}{1 + C_1 K_a}$ | | | |
| First derivative | $\frac{d\theta}{dT} = -\frac{\Delta H^{\circ}}{RT^2} \theta (1 - \theta)$ | g curves | | | |
| Second derivative | $\frac{\mathrm{d}^2 \theta}{\mathrm{d}T^2} = \frac{\Delta H^{\circ}}{RT^3} \theta (1 - \theta) \left[2 \right]$ | $+\frac{\Delta H^{\circ}}{RT}(1-2\theta)$ | | | |
| $\theta_{\rm max}(\theta-T)^{\rm a}$ | $\theta_{\max}(\theta-T)=0.5+B$ | | | | |
| First derivative | $\frac{d\theta/d(1/T)}{d\theta} = \frac{\Delta H^{\circ}}{R} \theta(1-\theta)$ | C | | | |
| Second derivative | $\frac{\mathrm{d}^2 \theta}{\mathrm{d}(1/T)^2} = \left(\frac{\Delta H^{\circ}}{R}\right)^2 \theta (1$ | $(1-\theta)(1-2\theta)$ | | | |
| $\theta_{\rm max}(\theta-1/T)$ | $\theta_{\rm max}(\theta - 1/T) = 0.5$ | | | | |
| a $(R = RT / \Lambda R$ | H°) | | | | |

^a $(B = RT_{\rm m}/\Delta H^{\circ})$

2.1. Comparison of spectroscopic and calorimetric melting profiles

It is important to consider relationships between melting temperatures from spectroscopic and calorimetric experiments. Differential scanning calorimetry (DSC) provides direct, model-independent measurements of energetics for helix-random coil transitions [5,12]. Specifically, assumption of two-state transition is unnecessary to obtain meaningful thermodynamic data (ΔH and ΔS). DSC measures "excess" molar heat capacity, $\Delta C_{\rm p}^{\rm exc}$, as a function of temperature for duplex to single strand transitions. The area below the excess heat capacity curve gives us calorimetric transition enthalpy, $\Delta H_{\rm DSC}$. This can be represented in differential form.

$$\Delta C_{\rm p}^{\rm exc} = \left(\frac{\partial \Delta H}{\partial T}\right)_{\rm p}.\tag{15}$$

If we assume temperature-independent enthalpy and a strictly two-state nature of the transition, the enthalpy change of a DSC sample is proportional to fraction of melted duplexes,

$$\Delta C_{\rm p}^{\rm exc} = \left(\frac{\partial \left(\theta \times \Delta H_{\rm DSC}\right)}{\partial T}\right)_{\rm p} = \Delta H_{\rm DSC} \left(\frac{\partial \theta}{\partial T}\right)_{\rm p}.\tag{16}$$

Eq. (16) reveals that the calorimetric melting temperature, which is typically read at maximum of $\Delta C_{\rm p}^{\rm exc}$ vs. temperature curve, is equivalent to $T_{\rm m}(\theta-T)$. Therefore, if the two-state assumption is reasonable, DSC melting temperature could be corrected to $T_{\rm m}({\rm half})$ using Eq. (10).

2.2. Automatic selection of melting curve baselines

To calculate the plot of θ versus temperature shown in Fig. 1, the lower baseline $A_{\rm L}$ and the upper baseline $A_{\rm LL}$ must be drawn. Sloping baselines are caused by increased unstacking of bases in both single-stranded and doublestranded molecules [7,9]. Experimental errors contributing to sloping baselines include drift of lamp intensity, sample evaporation, cuvette expansion, and sensitivity of detector to temperature changes when the detector is in close proximity to Peltier cuvette holder. Sloping baselines are usually least-squared fit to linear regions of absorbance versus temperature plots (see Fig. 2). Since the beginning and the end of melting transition are not always apparent, selection of baselines is somewhat arbitrary and it is usually done manually. We propose new algorithm, the second derivative method, which can be programmed and executed automatically.

Because melting curves are linear in baseline regions, the first derivative of melting curve, dA/dT, is constant and the second derivative, dA^2/dT^2 , is zero in these regions. In contrast, slopes of melting curves exhibit significant increases and decreases within melting transition where dA^2/dT^2 values are generally not zero. This difference in

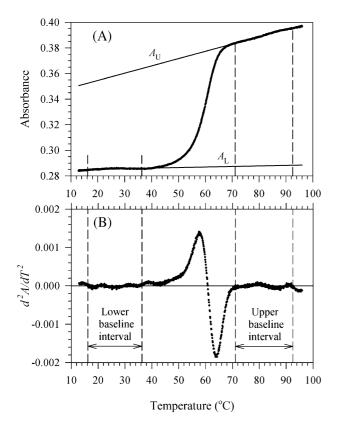


Fig. 2. (A) UV melting profile of d(TATGTATATTTTGTAATCAG) duplex in 1M NaCl-phosphate buffer [8], pH 7.0, C_t =2 μ M. (B) Linear regions for baseline fit were selected using the second derivative plot.

values of the second derivative can be utilized for baseline selection. Fig. 2 shows that global maximum and minimum of the second derivative melting curve is observed within melting transition. The lower baseline is obtained from region early in the melting curve, that is, before the second derivative plot reaches global maximum. The region used for the lower baseline fit is linear with zero $\mathrm{d}A^2/\mathrm{d}T^2$ values. The upper baseline is analogously selected in the region after global minimum of the second derivative plot where $\mathrm{d}A^2/\mathrm{d}T^2$ values are zero.

Because absorbance values contain experimental errors, melting curves need to be smoothed before the second derivative can be calculated. Ideal smoothing procedure removes high-frequency noise present in absorbance values without disturbing overall melting profile. Noise frequency can be estimated from Fourier transformation of absorbance data. We routinely apply Nearly Equal Ripple Filter [13], which smoothes melting curves effectively. The filter is based on running average algorithm where weighting coefficients are adjusted based on noise bandwidth. Melting curves can be smoothed with parameters of 0.03-0.05, 0.05 and 50 db for passband edge, band width and stop band loss [13], respectively. Experimental errors and smoothing artifacts cause small deviations of dA^2/dT^2 values from zero in linear regions. Large deviations of dA^2/dT^2 from zero indicate either insufficient smoothing or nonlinearity. Linear regions for baseline fit are between the first and the last temperature where the second derivative of melting curve crosses zero value (see Fig. 2). This method provides good estimates of baselines and successfully resolves the beginning and the end of melting transition; however, it fails to determine baselines when a part of melting transition is missing.

2.3. Effects of sloping linear baselines on analysis of melting curves

Melting temperatures are occasionally estimated from maxima of $\mathrm{d}A/\mathrm{d}T$ derivative melting curves, which are widely available in spectroscopic software. It would be useful to know the conversion function between $\mathrm{d}A/\mathrm{d}T$ and $\mathrm{d}\theta/\mathrm{d}T$ melting curves. The following equations relate absorbance, fraction of broken base pairs, and linear baselines,

$$A = \theta(A_{\rm U} - A_{\rm L}) + A_{\rm L} \tag{17}$$

$$A_{\rm L} = a_{\rm L} + b_{\rm L}T\tag{18}$$

$$A_{\rm U} = a_{\rm U} + b_{\rm U}T\tag{19}$$

where a_L , b_L , a_U , b_U are intercepts and slopes of lower and upper baselines, respectively.

Combining derivative of Eq. (17) with Eqs. (18) and (19) yields relationship between both derivative melting curves,

$$\frac{\mathrm{d}A}{\mathrm{d}T} = \frac{\mathrm{d}\theta}{\mathrm{d}T} \left[T(b_{\mathrm{U}} - b_{\mathrm{L}}) + a_{\mathrm{U}} - a_{\mathrm{L}} \right] + \theta(b_{\mathrm{U}} - b_{\mathrm{L}}) + b_{\mathrm{L}} \quad (20)$$

The second derivative is calculated to find maximum of dA/dT versus temperature plot,

$$\frac{d^{2}A}{dT^{2}} = \frac{d^{2}\theta}{dT^{2}} \left[T(b_{U} - b_{L}) + a_{U} - a_{L} \right] + 2 \frac{d\theta}{dT} (b_{U} - b_{L}) \quad (21)$$

If slopes of lower and upper baselines are same ($b_U=b_L$), then Eq. (21) reduces to,

$$\frac{\mathrm{d}^2 A}{\mathrm{d}T^2} = \frac{\mathrm{d}^2 \theta}{\mathrm{d}T^2} (a_{\mathrm{U}} - a_{\mathrm{L}}). \tag{22}$$

Eq. (22) indicates that maxima of dA/dT and $d\theta/dT$ melting curves are at identical temperature in this case. However, if slopes of baselines are different, the maxima of both derivative melting curves are not generally at the same temperature because the following relationship holds for dA/dT maximum,

$$\frac{\mathrm{d}^2 A}{\mathrm{d}T^2} = \frac{\mathrm{d}^2 \theta}{\mathrm{d}T^2} \left(T + \frac{a_{\mathrm{U}} - a_{\mathrm{L}}}{b_{\mathrm{U}} - b_{\mathrm{L}}} \right) + 2 \frac{\mathrm{d}\theta}{\mathrm{d}T} = 0 \tag{23}$$

The Eqs. (20)–(23) show the importance of correct analysis of melting curves. Baselines should be subtracted, so that the proper melting temperature, $T_{\rm m}({\rm half})$, is determined.

3. Experimental results

 $T_{\rm m}$ discrepancies predicted from Eq. (7), Eqs. (10) and (11) were compared with experimental melting data. Melting profiles for duplex d(TGGCGAGCAC) were analyzed and are shown in Fig. 1. The transition enthalpy of this duplex is estimated to be -323 kJ/mol using the nearest-neighbor model and unified parameter set [11]. Therefore, ratio B has a value of $8.3115 \times (273.15 + 56.6)$ $-323,000 = -8.49 \times 10^{-3}$. Using this value with the equations from Table 1, maxima $\theta_{\rm max}(\theta-T)$ and $\theta_{\rm max}(\theta-1/T)$ of derivative melting curves are predicted to be 0.574 and 0.586, respectively. These values are in good agreement with measured experimental values of 0.568 and 0.584. Melting temperatures obtained from maxima of derivative melting curves, $T_{\rm m}(\theta-T)$ and $T_{\rm m}(\theta-1/T)$, are shown in Fig. 1C and D. These temperatures were corrected to the temperature of transition midpoint; $T_{\rm m}$ (half) values of 55.4 and 55.5 °C, calculated from the Eqs. (10) and (11), respectively, are consistent with the experimental value of 55.4 °C.

This analysis was repeated for a set of 14 duplex DNAs, which were studied in our laboratory [8]. Duplexes ranged in length from 15 to 40 base pairs and in G•C base pair content from 30% to 73%. Experimental $T_{\rm m}$ differences were compared with predicted values. Results are presented in Table 4. The predicted $T_{\rm m}$ differences (last column of Table 4) are in reasonable agreement with experimentally measured $T_{\rm m}$ differences (column 7 of

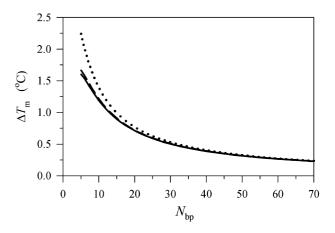


Fig. 3. Differences between the $T_{\rm m}(\theta-T)$, $T_{\rm m}(\theta-1/T)$, and $T_{\rm m}({\rm half})$ temperatures, which were predicted from equations in Table 1 are plotted as a function of oligomer length, $N_{\rm bp}$. The differences of $T_{\rm m}(\theta-T)-T_{\rm m}({\rm half})$ (solid line), $T_{\rm m}(\theta-1/T)-T_{\rm m}({\rm half})$ (dotted line) for non-self-complementary oligomers and the difference $T_{\rm m}(\theta-T)-T_{\rm m}({\rm half})$ for self-complementary oligomers (dash line) are shown. Results are calculated for a typical DNA duplex oligomer with 50% G+C base pair content, at $C_{\rm t}$ =1 μ M and in 1M Na $^+$ buffer.

Table 4). However, the predicted $\Delta T_{\rm m}$ values are smaller by 0.3–0.4 °C on average. Although this residual inaccuracy is small and within expected experimental error of melting temperatures (0.3 °C), the result suggests that Eq. (10) may not account for the entire experimentally observed difference between $T_{\rm m}(\theta-T)$ and $T_{\rm m}({\rm half})$ values. The origin of this residual inaccuracy is uncertain. $T_{\rm m}$ corrections presented here were developed assuming two-state melting transitions. Differences between predicted and experimental $\Delta T_{\rm m}$ s increase with $N_{\rm bp}$ (see Table 4). Because deviations of duplex melting transitions from the simple two-state model are known to increase with duplex

Comparison of experimentally measured and predicted discrepancies in melting temperatures (°C)

| DNA duplex sequence (5' to 3') | $N_{\rm bp}$ | f(G-C) | Predict. $\theta_{\text{max}}(\theta - T)^{\text{a}}$ | Exper. $T_{\rm m}$ (half) | Exper. $T_{\rm m}(\theta - T)$ | Exper. $\Delta T_{\rm m}^{\ \ b}$ | Predict. $\Delta T_{\rm m}^{\ \ c}$ |
|--|--------------|--------|---|---------------------------|--------------------------------|-----------------------------------|-------------------------------------|
| TTCTACCTATGTGAT | 15 | 0.33 | 0.577 | 53.7 | 55.0 | 1.3 | 0.9 |
| GCAGTGGATGTGAGA | 15 | 0.53 | 0.577 | 63.3 | 64.4 | 1.1 | 0.9 |
| CAGCCTCGTCGCAGC | 15 | 0.73 | 0.578 | 72.0 | 72.9 | 0.9 | 0.9 |
| TGATTCTACCTATGTGATTT | 20 | 0.30 | 0.579 | 64.4 | 65.2 | 0.8 | 0.7 |
| AGCTGCAGTGGATGTGAGAA | 20 | 0.50 | 0.579 | 73.1 | 74.1 | 1.0 | 0.7 |
| CAGCCTCGTTCGCACAGCCC | 20 | 0.70 | 0.580 | 78.1 | 79.5 | 1.4 | 0.7 |
| GTTCTATACTCTTGAAGTTGATTAC | 25 | 0.32 | 0.581 | 67.7 | 68.9 | 1.2 | 0.6 |
| CTGGTCTGGATCTGAGAACTTCAGG | 25 | 0.52 | 0.581 | 75.6 | 76.6 | 1.0 | 0.6 |
| CAGTGGGCTCCTGGGCGTGCTGGTC | 25 | 0.72 | 0.581 | 83.4 | 84.9 | 1.5 | 0.6 |
| CTTAAGATATGAGAACTTCAACTAATGTGT | 30 | 0.30 | 0.581 | 71.8 | 72.8 | 1.0 | 0.5 |
| AGTCTGGTCTGGATCTGAGAACTTCAGGCT | | 0.50 | 0.581 | 80.6 | 81.5 | 0.9 | 0.5 |
| GACCTGACGTGGACCGCTCCTGGGCGTGGT | | 0.70 | 0.582 | 86.4 | 87.3 | 0.9 | 0.5 |
| GCAATAGAAAGAGGAAATAATAGTTTTATATTCGACCTAG | | 0.30 | 0.583 | 75.4 | 76.2 | 0.8 | 0.4 |
| AGCTGACGCCAAGTCCAAATCTAACCACATGCAAGACACG | 40 | 0.50 | 0.583 | 84.5 | 85.3 | 0.8 | 0.4 |

^a Predicted from the Eq. (7)

b Determined from experimental melting profiles, $\Delta T_{\rm m} = T_{\rm m}(\theta - T) - T_{\rm m}({\rm half})$, in 1 M Na⁺ buffer, $C_{\rm t} = 2~\mu{\rm M}$ [8].

^c Predicted from the Eq. (10) and experimental $T_{\rm m}(\theta-T)$. Transition enthalpies were estimated from the nearest-neighbor model and unified parameters [11].

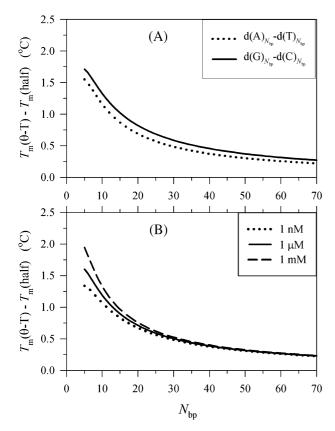


Fig. 4. Differences between the $T_{\rm m}(\theta-T)$ and $T_{\rm m}({\rm half})$ melting temperatures, which were predicted from the equations in Table 1 are plotted as a function of oligomer length, $N_{\rm bp}$. Results are calculated for DNA duplexes in 1M Na⁺ buffer using the nearest-neighbor model and unified parameters [11]. (A) Effects of G'C base pair content on $T_{\rm m}$ differences. Values of $\Delta T_{\rm m}$ for $({\rm dG})_{N_{\rm bp}} - ({\rm dC})_{N_{\rm bp}}$ duplexes (solid line) and $({\rm dA})_{N_{\rm bp}} - ({\rm dT})_{N_{\rm bp}}$ duplexes (dotted line) are shown at $C_{\rm t}$ = 1 μ M. (B) Effects of total single strand DNA concentrations from 1 nM to 1 mM on $T_{\rm m}$ differences calculated for a typical 50% G•C DNA duplex.

length, we speculate that residual inaccuracy in predicted $\Delta T_{\rm m}$ values may be caused by minor deviations from two-state approximation.

It is useful to know the range of $\Delta T_{\rm m}$ values that arise from use of different analysis methods and if these values vary with oligomer length, G-C base pair content, the type of hybridization, and nucleic acid concentration. We therefore predicted $T_{\rm m}$ discrepancies for most commonly studied bimolecular hybridizations using expressions from Table 1 as well as the Eqs. (10) and (11). Unified parameters [11] suggest that transition enthalpies and entropies of the representative DNA duplex with 50% G•C content depend on the number of base pairs, $N_{\rm bp}$, in the following fashion, $\Delta H^{\circ} = -34.4(N_{\rm bp} - 1) + 10^{\circ} \text{ kJ/mol}$ and $\Delta S^{\circ} =$ $-92.5(N_{\rm bp}-1)+5.4$ J/(mol K). Using these relationships and assuming that both single strands are present in equal concentrations, $T_{\rm m}$ discrepancies were predicted. Results displayed in Fig. 3 show that $T_{\rm m}(\theta - 1/T) - T_{\rm m}({\rm half})$ difference (dotted line) is larger than $T_{\rm m}(\theta-T)-T_{\rm m}({\rm half})$ difference (solid line), in particular for DNA oligomers shorter than 20 base pairs. Comparing equilibrium of self-complementary oligomers (dash line) with equilibrium of non-self-complementary oligomers (solid line), $T_{\rm m}(\theta-T)-T_{\rm m}({\rm half})$ differences are very similar. In both kinds of hybridizations, significant dependence of $\Delta T_{\rm m}$ on the length of duplexes is observed. The $\Delta T_{\rm m}$ values are larger for shorter oligomers. They increase from 0.5 °C for a representative 30-mer to 1.5 °C for 6-mer. Fig. 4 shows effects of G•C base pair content and DNA concentration on $\Delta T_{\rm m}$ values. Duplex DNAs with both high and low G•C contents exhibit similar $\Delta T_{\rm m}$ values, i.e., the $T_{\rm m}(\theta-T)-T_{\rm m}({\rm half})$ difference is weakly dependent on oligomer $f({\rm G} {\cdot} {\rm C})$. Effects of DNA concentrations are significant for oligomers shorter than 20 base pairs. We observe that the $T_{\rm m}(\theta-T)-T_{\rm m}({\rm half})$ difference increases as DNA concentration increases.

4. Conclusions

General equations for correction of $T_{\rm m}$ values obtained using different methods of data analysis were derived and experimentally verified. For cases where a two-state model holds, these equations can be used to correct inconsistencies between published data sets, permitting improved analysis and collection of melting data from multiple sources. New method of melting curve analysis is suggested which allows automatic selection of sloping baselines.

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